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Anomaly induced effects in a magnetic field

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Abstract

We consider a modification of electrodynamics by an additional light massive vector field, interacting with the photon via Chern-Simons-like coupling. This theory predicts observable effects for the experiments studying the propagation of light in an external magnetic field, very similar to those, predicted by theories of axion and axion-like particles. We discuss a possible microscopic origin of this theory from a theory with non-trivial gauge anomaly cancellation between massive and light particles (including, for example, millicharged fermions). Due to the conservation of the gauge current, the production of the new vector field is suppressed at high energies. As a result, this theory can avoid both stellar bounds (which exist for axions) and the bounds from CMB considered recently, allowing for positive results in experiments like ALPS, LIPPS, OSQAR, PVLAS-2, BMV, Q&A, etc.

1 Introduction

A number of experiments, studying the properties of vacuum in a strong external magnetic field (in particular the propagation of light in such a field), have been performed over the recent years, or are being in operation at the moment [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. One of the main motivations for these experiments is the search for a hypothetical particle, the axion, which was originally predicted [12, 13] by the

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Peccei-Quinn mechanism of solving the "strong CP problem" in QCD [14, 15]. The main property of this particle from the laboratory experiments' point of view is the coupling of the axion field $a(x)$ to the photon via the interaction term $\frac{1}{4}a(x)F\tilde{F} = a(x)(\vec{E}\cdot\vec{H})$. More generally, axion-like-particles (ALP) are pseudo-scalars which interact with $F\tilde{F}$,[1] but may not have other properties of the QCD axion needed to solve the strong CP problem.

In [16] and [17, 18] another class of effective theories, which also predict effects in the presence of electric and magnetic fields, having completely different particle physics motivation, were considered. The coupling of axion to $F\tilde{F}$ appears in general from chiral *gauge anomalies*, that may be present in theories with chiral couplings of fermions to a gauge field. Although gauge anomaly makes theory inconsistent, the theories with several chiral fields may be well-defined if the overall anomaly cancels and the overall gauge current is conserved. The fact that the net vacuum current is conserved does not mean that it is zero. If it is not, the non-trivial anomaly cancellation between heavy and light fields may give rise to experimental signatures of the heavy fields which, due to the topological nature of anomalies, are not suppressed by the mass of the heavy fermions and, therefore, may be observed at low energies [18]. For theories with non-trivial electromagnetic anomaly cancellations, such effects will be proportional to $F\tilde{F}$.

In this paper, we discuss in more details the theory considered in [16] and, in particular, show how the creation of the new light particles, present there, may be suppressed with energy and, therefore, the parameters of the theory may not be constrained by the stellar bounds, which severely restrict the parameter space of axions (for a recent review, see e.g. [19]). We describe the possible origin of the effective Lagrangian of [16] and show that it may appear, for example, from a theory of millicharged fermions, interacting with the photon and a "paraphoton" similar to the theory considered in [20, 21]. This allows to have the effects of vacuum dichroism and birefringence [22, 23] and "shining light through the wall" [24], observable in experiments like PVLAS [25], OSQAR [7], ALPS [26, 8], BMV [9, 10], LIPPS [11] avoiding not only the stellar constraints for the Chern-Simons like interaction of the paraphoton, but also the bounds on the parameters of millicharged particles discussed in [27].

The organization of the paper is as follows. In Section 2, we review the effective field theory of [16] that introduces a new massive vector boson, coupled to the photon via a Chern-Simons term, and study the propagation of light in a magnetic field. At low energies, this theory behaves like a theory of ALP corresponding to the longitudinal component of the new gauge field. In Section 3, we modify it at high energies, so that the gauge boson couples to a conserved current, suppressing the production of its longitudinal component in stars. This modification is parametrized by effective non-local terms. We then provide an example of a microscopic theory

[1] We denote by $\tilde{F}$ the dual field strength: $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$.

reproducing these terms, using millicharged fermions. We also analyze the various experimental constrains and deduce the allowed parameter space that leaves open the possibility of detecting interesting effects in forthcoming experiments, such as measuring dichroism and birefringence of light and photon regeneration in the presence of strong magnetic fields. Section 4 contains our concluding remarks and a discussion on possible derivation of our model from D-brane constructions.

2 Effective theory at low energies and propagation of light in the magnetic field.

We start by reminding the model considered in [16]. We consider the effective theory of two vector fields: the usual photon A_μ and a massive field B_μ (paraphoton):[2]

$$S_{low} = \int d^4x \left(-\frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{m_B^2}{2}(D_\mu\theta)^2 + \frac{m_\gamma^2}{2}(D_\mu\chi)^2 + 2\boldsymbol{\kappa} D\chi \wedge D\theta \wedge F_A \right) \quad (1)$$

where $D\theta = d\theta + B$, $D\chi = d\chi + A$ and $\boldsymbol{\kappa}$ is a dimensionless coupling. The Stuckelberg field θ makes the action explicitly gauge invariant under $U(1)_B$ gauge transformations, provided that $\theta \to \theta - \alpha$ when $B_\mu \to B_\mu + \partial_\mu\alpha$. The field B_μ is therefore massive with mass m_B. Another Stuckelberg field χ, transforming as $\chi \to \chi - \lambda$ when $A_\mu \to A_\mu + \partial_\mu\lambda$, restores the gauge invariance with respect to the $U(1)_A$ gauge group. This means that the photon A_μ is also massive with mass m_γ. This becomes explicit in the *unitary gauge* ($\theta = 0$, $\chi = 0$):

$$S_{low,\,unitary} = \int d^4x \left(-\frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{m_B^2}{2}B_\mu^2 + \frac{m_\gamma^2}{2}A_\mu^2 + 2\boldsymbol{\kappa} A \wedge B \wedge F_A \right) \quad (2)$$

Notice, that sending $m_\gamma \to 0$ (and making the field χ non-dynamical) will make A_μ massless and add the constraint $F_A \wedge F_B = 0$ (with the field χ being a Lagrange multiplier). To make the analysis simpler, we will assume massless photon and the action (2) supplemented by this constraint, although our analysis remains unchanged in the case of finite but small enough m_γ. The Chern-Simons (CS) term $\boldsymbol{\kappa} A \wedge D\theta \wedge F_A$ remains gauge invariant in this case. Notice, that in the theory (1) the field B_μ couples to a non-conserved current $(J_B^\mu)_{low}$:

$$(J_B^\mu)_{low} = \frac{\delta S_{low}}{\delta B_\mu} = \boldsymbol{\kappa}\epsilon^{\mu\nu\lambda\rho}A_\nu F_{\lambda\rho} \quad ; \quad \partial_\mu (J_B^\mu)_{low} = \frac{\boldsymbol{\kappa}}{2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} \quad (3)$$

The conserved (Nöther) current is a linear combination of $(J_B^\mu)_{low}$ and $\partial_\mu\theta$. As we will show below, the property (3) implies that in the theory (1) the longitudinal part

[2] To simplify the notations and avoid proliferation of non-essential ϵ-tensors, we will often use differential forms, such as $A \wedge D\theta \wedge F_A$ instead of $\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}A_\mu(\partial_\nu\theta + B_\nu)F_{\lambda\rho}$ and $F \wedge F$ instead of $\frac{1}{4}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$.

of the massive vector field B_μ behaves as a massive axion with the mass m_B and the coupling constant $m_B/\boldsymbol{\kappa}$.

2.1 Propagation of light in a magnetic field

Let us consider the equations of motion of theory (1) for the case of the propagation of photons in strong external magnetic field. We consider light propagation along the z-axis in the magnetic field $\boldsymbol{H_0} = F_{yz} = F_{23}$, pointing in x-direction. Under the condition $F_A \wedge F_B = 0$, the CS term does not depend on the choice of the background vector potential. For simplicity we choose the background vector potential, corresponding to the field $\boldsymbol{H_0}$, in the form $\bar{A}_y = -z\boldsymbol{H_0}$. Working in the unitary gauge (i.e. $\theta = 0$) for the B-field, we obtain the following system of equations:

$$\partial_\nu F^{\mu\nu} = -\boldsymbol{\kappa}\epsilon^{\mu\nu\lambda\rho}\Big(2B_\nu F_{\lambda\rho} - A_\nu F^B_{\lambda\rho}\Big) \tag{4}$$

$$\partial_\nu F_B^{\mu\nu} - m_B^2 B^\mu = \boldsymbol{\kappa}\epsilon^{\mu\nu\lambda\rho} A_\nu F_{\lambda\rho} \tag{5}$$

$$\epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F^B_{\lambda\rho} = 0 \tag{6}$$

One can construct the solution of these equations as perturbations in small parameter $\boldsymbol{\kappa}$ around the "zeroth order approximation": $A_\mu^{(0)} = a_\mu e^{i\omega(t-z)}$, $B_\mu^{(0)} = 0$.

A simple analysis shows that only the longitudinal part of B_μ (i.e. longitudinal electric field F^B_{0z}) interacts with the propagating photon, with polarization parallel to the external magnetic field. To demonstrate the similarities and differences between this model and the axion, we revert from eqs. (4-5) to the system, involving the longitudinal degree of freedom of the B-field and the photon A_x.[3] Namely, one can represent the field B_μ in terms of one scalar degree of freedom, ϕ:

$$B_\mu = \frac{1}{m_B}\partial_\mu\phi + \delta_{\mu 0}\tilde{B}_0 \quad \text{i.e.} \quad \begin{cases} B_z = \dfrac{\partial_z\phi}{m_B} \\ B_0 = \dfrac{\partial_0\phi}{m_B} + \tilde{B}_0 \end{cases} \tag{7}$$

Notice that there is a residual uncertainty in the definition of ϕ: one can simultaneously shift $\phi \to \phi + \beta(t)$ and $\tilde{B}_0 \to \tilde{B}_0 - \frac{\dot{\beta}(t)}{m_B}$. Clearly, $F^B_{0z} = -\partial_z\tilde{B}_0$.

To establish the connection between ϕ and $\tilde{B}_0$ we consider the Maxwell's equation:

$$i\omega F^B_{0z} + m_B\partial_z\phi = -2\boldsymbol{\kappa}\bar{A}_y(z)(i\omega A_x)\ , \tag{8}$$

which gives

$$\tilde{B}_0 = \frac{m_B}{i\omega}\phi + 2\boldsymbol{\kappa}\int^z \bar{A}_y(z')A_x(z')dz'\ . \tag{9}$$

[3] We choose the gauge in which the plane wave, with polarization vector parallel to the external magnetic field is described by the potential $A_x(t,z)$.

Taking the derivative of eq. (5) we arrive to the first class constraint which massive vector fields B_μ should obey

$$\partial_\mu B^\mu = -\frac{\boldsymbol{\kappa}}{2m_B^2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho} = -\frac{4\boldsymbol{\kappa}\boldsymbol{H_0}(i\omega A_x)}{m_B^2} \tag{10}$$

Putting relations (7), (9) into the constraint (10) we obtain

$$(\Box + m_B^2)\phi = -\frac{4\boldsymbol{\kappa}\boldsymbol{H_0}i\omega A_x}{m_B} - 2i\boldsymbol{\kappa}m_B\omega\int^z \bar{A}_y(z')A_x(z')dz' \tag{11}$$

Next, we analyze the e.o.m. for the field A_x:

$$\begin{aligned}\Box A_x = \frac{4\boldsymbol{\kappa}\boldsymbol{H_0}i\omega}{m_B}\phi + \frac{4\boldsymbol{\kappa}\boldsymbol{H_0}m_B}{i\omega}\phi + 8\boldsymbol{\kappa}^2\boldsymbol{H_0}\int^z \bar{A}_y(z')A_x(z')dz' \\ + \frac{2\boldsymbol{\kappa}m_B}{i\omega}\partial_z\phi\bar{A}_y + 4\boldsymbol{\kappa}^2\bar{A}_y^2A_x\end{aligned} \tag{12}$$

The propagation of light, polarized perpendicularly to the magnetic field (component A_y), is not modified in this theory, as follows trivially from the structure of currents in the right hand side of eqs. (4)–(5).

Let us compare eqs. (11)–(12) to the pure axion case:

$$\begin{aligned}\partial_\nu F^{\mu\nu} &= \frac{1}{M}\epsilon^{\mu\nu\lambda\rho}(\partial_\nu\phi)F_{\lambda\rho} \\ (\Box + m_a^2)\phi &= \frac{1}{4M}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\end{aligned} \tag{13}$$

For the propagation of linearly polarized light, parallel to the magnetic field $\boldsymbol{H_0}$ equations (13) reduce to

$$\begin{aligned}(\Box + m_a^2)\phi &= -\frac{2i\omega}{M}A_x\boldsymbol{H_0} \\ \Box A_x &= \frac{2i\omega\boldsymbol{H_0}}{M}\phi\end{aligned} \tag{14}$$

We see that under the identification

$$\frac{1}{M} \leftrightarrow \frac{2\boldsymbol{\kappa}}{m_B} \quad ; \quad m_a \leftrightarrow m_B \tag{15}$$

equations (11)–(12) reduce to (14) plus some additional (non-local) terms, depending on the background potential $\bar{A}_y$.[4] These terms, however, are suppressed (as compared to those, depending on the field $\boldsymbol{H_0}$). The easiest way to see it, is to send $\boldsymbol{\kappa} \to 0$, while keeping the ratio $\boldsymbol{\kappa}/m_B = \frac{1}{2M}$ *fixed*. Thus, equations (11)–(12) are reduced into those of massless axion. This leads to the dichroism $\sim \frac{\boldsymbol{\kappa}^2}{m_B^2}\boldsymbol{H_0}^2L^2$, where L is the distance traveled by the light. For finite m_B, corrections suppressed by powers of $\frac{m_B}{\omega}$ will appear.

[4]The term $\frac{4\boldsymbol{\kappa}\boldsymbol{H_0}m_B}{i\omega}\phi$ in eq. (12) is subleading, as compared to the first term and does not change the results of our analysis in any essential way.

3 Possible microscopic origin of the theory.

3.1 Decoupling of the longitudinal polarization at high energies

In quantum electrodynamics, if one adds a small photon mass m_γ, all the processes, involving the third (longitudinal) degree of freedom, are suppressed at energies $E \gg m_\gamma$ as $\frac{m_\gamma}{E}$. This is due to the fact that the electromagnetic field couples to a conserved current, which is a consequence of gauge invariance of the theory. On the other hand, if the current is not conserved, at high energies processes involving the longitudinal polarization of the vector boson are equivalent to those involving the longitudinally coupled scalar due to the so called *Goldstone boson equivalence theorem* [28]

This is what happens in theory (1). Although the theory is gauge invariant under the $U(1)_B$ gauge symmetry, it is realized by simultaneous gauge transformations of the B-field and the Stuckelberg field θ. As we saw in Section 2.1, the field B_μ couples to the non-conserved current (3) and therefore its longitudinal polarization behaves as an axion (for $\omega \gg m_B$).

However, the theory (1) is an effective field theory, valid up to a certain energy scale μ. In many cases it naturally happens that for $E \gtrsim \mu$ the theory gets modified in such a way that the current, to which the B_μ-field couples becomes conserved. Then, all processes involving emission or absorption of the longitudinal polarization of B_μ are suppressed as $\left(\frac{m_B}{\omega}\right)^2$. On the other hand, we are interested in the situation, where the field B_μ can be produced at low laboratory energies. This puts restriction on its mass to be $m_B \lesssim \omega_{\rm lab} \sim 1$ eV. Then at stellar energies $\omega_{\rm star} \sim 1$ keV one obtains a suppression of *at least* $\sim (\omega_{\rm lab}/\omega_{\rm star})^2 \sim 10^{-6}$. For smaller values of m_B the suppression is even stronger. The theory of transverse B_μ-field with CS interaction does not resemble the theory of axion anymore. For example, the production of this field due to the CS interaction is strongly suppressed by the small value of the dimensionless CS coupling $\boldsymbol{\kappa}$.

To illustrate this idea, assume that in the theory there is an additional particle with mass m_Ψ, interacting with the fields of the theory (1) and giving rise to an effective action of the following (schematic) form:

$$S = \int d^4x \left(-\frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{m_B^2}{2}(D\theta^2) + \boldsymbol{\kappa} A \wedge B \wedge F_A \right. \\ \left. + \boldsymbol{\kappa}\, \theta \frac{m_\Psi^2}{\Box + m_\Psi^2}(F\tilde{F}) - \boldsymbol{\kappa}\, (\partial_\mu B^\mu) \frac{1}{\Box + m_\Psi^2}(F\tilde{F}) \right) \tag{16}$$

In the next subsection we will present an example of a renormalizable field theory, which has such properties (recall that we always add to this theory the constraint $F_A \wedge F_B = 0$ to make it gauge invariant). At *low energies* (for $\omega < m_\Psi$) one obtains the action (1) (formally taking $m_\Psi \to \infty$).

	Ψ_1		Ψ_2		Ξ_1		Ξ_2	
	ψ_L	ψ_R	ψ'_L	ψ'_R	χ_L	χ_R	χ'_L	χ'_R
$\mathrm{U}(1)_A$	e_1	e_1	e_2	e_2	$-e_3+\delta e$	e_3	e_3	$-e_3+\delta e$
$\mathrm{U}(1)_B$	q_1	$-q_1$	$-q_1$	q_1	q_2	q_2	q_2	q_2

Table 1: A simple choice of charges of fermions, which leads to the low-energy effective action (16). The charges are chosen in such a way that all gauge anomalies cancel. The cancellation of $U(1)_A^3$ and $U(1)_B^3$ anomalies happens for any value of e_i, q_i. Cancellation of mixed anomalies requires $\delta e = \frac{\boldsymbol{\kappa}}{4q_3e_3}$ where $\boldsymbol{\kappa}$ is related to e_i, q_i via (21).

At high energies ($\omega \gg m_\Psi$) the effective theory is of course non-local. We can neglect the interacting term proportional to θ in the action (16) and obtain

$$S(\omega \gg m_\Psi) \approx \int -\frac{1}{4}F_A^2 - \frac{1}{4}F_B^2 + \frac{m_B^2}{2}D_\mu\theta^2 + \boldsymbol{\kappa} A\wedge B\wedge F_A - \boldsymbol{\kappa}\,(\partial_\mu B^\mu)\frac{1}{\Box}(F\tilde{F}) \quad (17)$$

At these energies, the field B_μ couples to the *conserved* current J_B

$$J_B^\mu = \frac{\boldsymbol{\kappa}}{2}\epsilon^{\mu\nu\lambda\rho}A_\nu F_{\lambda\rho} - \boldsymbol{\kappa}\frac{\partial_\mu}{\Box}(F\tilde{F})\;. \quad (18)$$

Therefore, at energies $\omega \gg m_\Psi$ the production of the longitudinally polarized B_μ-field in theory (16) is suppressed. Of course, for $\omega > m_\Psi$ the current (18) should be computed directly in the microscopic theory producing the non-local terms in (16), containing additional particles, rather than in the non-local effective theory. Next we are going to present an example of such a microscopic theory.

3.2 Theory with millicharged fermions

Consider a theory with several sets of chiral fermions, whose masses are given by Yukawa interactions with Higgs fields Φ_1 and Φ_2:

$$\mathcal{L} = \sum_{i=1,2}\Big(i\bar{\Psi}_i\,\not{D}\,\Psi_i + f_i\bar{\Psi}_i\Phi_1\Psi_i\Big) + \Big(i\bar{\Xi}_i\,\not{D}\,\Xi_i + \lambda_i\bar{\Xi}_i\Phi_2\Xi_i\Big) + \text{h.c.} \quad (19)$$

The fermions are charged with respect to $U(1)_A\times U(1)_B$, with one of the possible choices of charges shown in Table 1. Integrating out the fermions for energies below their masses, one obtains "anomalous" (CS-like) terms in the effective action :[5]

$$S_{\text{cs}} = \int \frac{(e_1^2-e_2^2)q_1}{16\pi^2}\theta F_A\wedge F_A + \boldsymbol{\kappa} A\wedge B\wedge F_A \quad (20)$$

[5]Of course, integrating out these fermions, one obtains also terms leading to a renormalization of charges of the fields A_μ and B_μ, of the B_μ mass, possible generation of kinetic mixing between A_μ and B_μ, etc. However, in the case of chiral fermions, one expects additional contributions from "anomalous" (triangular) diagrams (shown in Figs. 1–2) (see e.g. [29]).

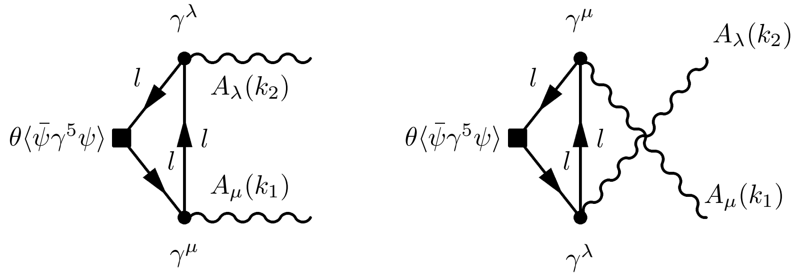

Figure 1: Anomalous contributions to the correlator $\langle\bar{\psi}\gamma^5\psi\rangle$.

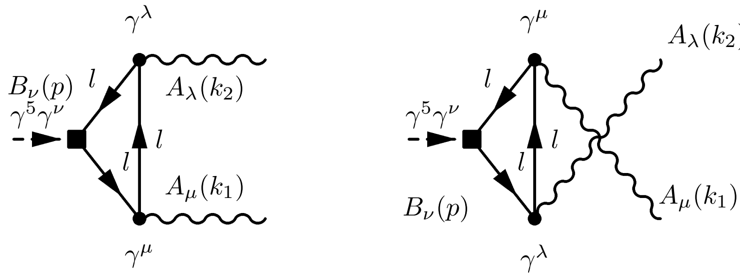

Figure 2: Two graphs, contributing to the Chern-Simons terms

Only fermions Ψ contribute to the term $\theta F_A \wedge F_A$ (fermions Ξ are vector-like with respect to the $U(1)_B$ gauge group and therefore do not interact with θ). The contribution to the CS term $A \wedge B \wedge F_A$ comes from both sets of fermions. The interaction of the B_μ-field with the photon is very weak due to the small value of $\boldsymbol{\kappa}$, therefore in the action (20) we omitted anomalous terms, containing more than one power of the B-field.

The relation between the coefficient $\boldsymbol{\kappa}$ in front of the CS term and the fermion charges is dictated by the gauge invariance of the action with respect to $U(1)_B$ gauge transformation

$$\boldsymbol{\kappa} = \frac{q_1(e_1^2 - e_2^2)}{16\pi^2} \tag{21}$$

The Higgs field Φ which provides mass to the fermions Ψ_i is charged with respect to the $U(1)_B$ with charge $2q_1$. If it acquires a vacuum expectation value (VEV) $|\Phi_1| = v_B$, one has $\Phi_1 = v_B e^{2iq\theta}$ and the Yukawa term for Ψ fields becomes $v_B \bar{\Psi}_i e^{2iq_1\theta\gamma^5}\Psi_i$. The Higgs then provides mass to the B_μ-field:

$$m_B = 2q_1 v_B \tag{22}$$

and Higgs's kinetic term becomes that of the field θ in action (1).

Let us take Yukawa couplings $f_i \ll \lambda_i$, so that fermions Ξ are much heavier that Ψ. Consider energies $m_\Psi < E < m_\Xi$. The resulting expression for the current J_B^μ is

given by the expression

$$J_B^\mu = \frac{\boldsymbol{\kappa}}{2}\epsilon^{\mu\nu\lambda\rho}A_\nu F_{\lambda\rho} - \boldsymbol{\kappa}\frac{\partial_\mu}{\Box}(F\tilde{F}) \tag{23}$$

which arises from the effective action

$$S_{eff,high} = \int \boldsymbol{\kappa}(\partial_\mu B^\mu)\frac{1}{\Box}F_A \wedge F_A + \boldsymbol{\kappa} A \wedge B \wedge F_A \tag{24}$$

(compare e.g. [30] where similar result for the massless chiral fermions is presented).

3.3 The parameters of the model

Next, we study the restrictions on the parameters of our model. First of all, notice that in order to expect positive outcome in laboratory experiments, which aim to measure dichroism and birefringence or photon regeneration (shining light through the wall), we need to have a mass of the B_μ-field to be within the energy reach of laboratory experiments:

$$m_B \lesssim \omega_{\rm lab} \sim 1 \text{ eV} \tag{25}$$

This means that

$$v_B \lesssim 1 \text{ eV}/q_1 \tag{26}$$

Notice, that the charges of fermions with respect to the "paraphoton" B_μ are not restricted and therefore one may have $q_1 \lesssim 1$. This will make v_B to be also in the sub-eV region.

When speaking about ALPs, it is convenient to characterize their interaction with the photon by the dimensionful coupling M (see eqs. (13)). It follows from eq. (15) that in terms of the parameters of our model $M \sim \frac{m_B}{\boldsymbol{\kappa}}$. Using then relations (21) and (22), we obtain the following expression for the interaction strength M in terms of the parameters of the microscopic model:[6]

$$M \sim \frac{v_B}{e_\Psi^2} \tag{27}$$

For our analysis of the classical equations of motion to remain valid, the tree-level unitarity bound for processes involving massive vector bosons should not be saturated at all relevant energies. The saturation of this bound is reached at energies $E \sim m_B/\boldsymbol{\kappa}$ which is again M.

There are various restrictions on the charges of new types of fermions with the electromagnetic field. First, laboratory bounds, coming from the contributions to the Lamb shift [31] and invisible orthopositronium decay [31] (based on the results of [32]) give $e_\Psi < 10^{-4}$. This translates into

$$M \gtrsim 10^8 v_B = 0.1 \text{ GeV}\left(\frac{v_B}{\text{eV}}\right) \tag{28}$$

[6]We consider the case when $e_1 \sim e_2 \sim e_\Psi$.

This bound is very weak. A stronger bound on the charges of millicharged fermions ($e_\Psi < 10^{-6}$) with sub-eV masses comes from the requirement that such fermions do not distort the CMB spectrum too much [27]. However, this restriction is not applicable in our case as the mass of our fermions $m_\Psi > 1$ eV (see discussion below).

The strongest bounds on charges of fermions with mass below ~ 30 keV comes from limiting the contribution of these particles to the energy transfer in stars [33] (see also [34]). This gives $e_\Psi < 10^{-14}$. This bound translates into the following lower bound on M:

$$M > 10^{28} v_B = 10^{19}\ \mathrm{GeV}\left(\frac{v_B}{\mathrm{eV}}\right) \tag{29}$$

For instance, if we take $M \sim 10^9$ GeV (the sensitivity expected to be reached by the OSQAR experiment [7]), this would require $v_B \sim 10^{-10}$. This would imply an extremely light B_μ field with $m_B \sim 10^{-10}$ eV and $\boldsymbol{\kappa} \sim 10^{-28}$ (which is still a possibility).

However, the mass m_B is not necessarily so small. In our model, the paraphoton field B_μ would acquire a kinetic mixing with the photon due to the loop corrections coming from light fermions. Therefore, the mechanism of additional suppression of the coupling of fermions with the photon in stars, similar to that proposed in [35], is possible. The restriction then becomes $e_\Psi \lesssim 10^{-14}\left(\frac{\omega_{\mathrm{star}}}{m_B}\right)^2$, i.e. the stellar bound of [33, 31] is weakened by *at least* six orders of magnitude. This would lead to the following bound

$$M \gtrsim 10^7\ \mathrm{GeV}\left(\frac{v_B}{\mathrm{eV}}\right)^5 \tag{30}$$

with $m_B \lesssim 1$ eV.

We are interested in the situation where at laboratory energies the effective action is given by (2). For this to be true, the mass m_Ψ of the fermions Ψ should be greater than laboratory energies ω_{lab}. At the same time, the mass should be lower than the stellar energies $\omega_{\mathrm{star}} \sim 1$ keV for (23) to be true. Therefore the fermion mass m_Ψ lies in the interval 1 eV $\lesssim m_\Psi \lesssim$ 1 keV. As we saw in Section 3.1, in the limit $m_\Psi/\omega_{\mathrm{star}} \to 0$ the massive vector field B_μ is coupled to a conserved current and therefore its production in stars is suppressed *at least* as $\left(\omega_{\mathrm{lab}}/\omega_{\mathrm{star}}\right)^2$. At higher energies ($\omega \sim \omega_{\mathrm{star}}$) additional contributions to the action (24) due to finite $m_\Psi/\omega_{\mathrm{star}}$ behave as $\left(m_\Psi/\omega_{\mathrm{star}}\right)^2$. These terms contribute to the divergence of the current J_B^μ:

$$\partial_\mu J_B^\mu \sim \boldsymbol{\kappa}\left(\frac{m_\Psi}{\omega_{\mathrm{star}}}\right)^2 \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \tag{31}$$

This gives an additional contribution to the production of the longitudinally polarized B-field, not suppressed by $(m_B/\omega)^2$. However, in comparison with the low-energy expression (3), there is an additional suppression $\left(m_\Psi/\omega_{\mathrm{star}}\right)^2$. For convenience, in what follows, we will characterize the interaction strength in terms of an energy

dependent coupling constant $M(\omega)$:

$$M(\omega) \sim \frac{m_B}{\boldsymbol{\kappa}} \left(\frac{\omega}{m_\Psi}\right)^2 \quad , \quad \omega > m_\Psi \tag{32}$$

To determine the admissible values of m_Ψ, let us assume that one of the laboratory experiments (light propagation in a magnetic field or regeneration experiments) had positive outcome. This determines the value of $M_{\rm lab} \sim m_B/\boldsymbol{\kappa}$. As a conservative estimate we take it to be equal to the laboratory lower bound, reported by PVLAS collaboration [6]: $M_{\rm lab} \gtrsim 2 \times 10^6$ GeV. This value is much lower than those coming from the astrophysical bounds: restriction from the helioseismology $M_\odot \gtrsim 2 \times 10^9$ GeV [36, 37] and restrictions from the horizontal branch (HB) stars: $M_{\rm HB} \gtrsim 10^{10}$ GeV (for a review see e.g. [34, 38, 19]). Demanding that the modification (32) at star energies $M(\omega_{\rm star})$ is greater than these astrophysical bounds (which we will collectively call $M_{\rm star}$), one arrives to the following restriction on the mass m_Ψ:

$$m_\Psi \lesssim \omega_{\rm star} \sqrt{\frac{M_{\rm lab}}{M_{\rm star}}} \tag{33}$$

As an estimate for $\omega_{\rm star}$ we take the plasma frequency in the stellar interim: $\omega_{\rm star} \sim 0.3$ keV for the Sun and $\omega_{\rm star} \sim 2$ keV for HB stars. As a result, we obtain $m_\Psi \lesssim 10$ eV using solar data or $m_\Psi \lesssim 20$ eV using restrictions from HB stars.

The recent bound from CAST collaboration [39, 40] $M_{\rm CAST} \gtrsim 10^{10}$ GeV would give the restriction on the mass of the fermions $m_\Psi \lesssim$ few eV. However, it should be noted that the present CAST bounds do not extend above axion masses ~ 0.02 eV. The model presented here allows $m_B \gtrsim 0.02$ eV (while still below $\omega_{\rm lab}$), therefore the CAST restrictions may not be applicable in our case.[7]

Additional restriction on the parameter space may come from the following fact. Integrating out fermions Ψ leads in general to terms suppressed by $1/m_\Psi$ (as in any renormalizable field theory [41]). At laboratory experiment energies with $\omega \sim 1$ eV, such contributions may still be significant. In particular, they contribute to the action the following terms (analogous to the Euler-Heisenberg corrections in quantum electrodynamics [42])

$$S_\Psi = \int d^4x \, \frac{2}{45} \frac{e_\Psi^4}{m_\Psi^4} \left(\frac{7}{16} (F\tilde{F})^2 + 4(F_{\mu\nu}^2)^2 \right) \tag{34}$$

These terms of course do not contribute to dichroism (as we consider $\omega < 2m_\Psi$), while their contribution to birefringence is given by

$$\beta_\psi \sim (\omega L) \frac{e_\Psi^4}{m_\Psi^4} \boldsymbol{H}_0^2 \tag{35}$$

[7] The CAST Phase II will test the range of masses 0.02 eV $< m_a <$ 1.1 eV [40] and will allow to probe further our model.

Taking $e_\Psi \sim 10^{-8}$, $m_\Psi \sim 1$ eV (to maximize the value of (35)), and typical values for $\omega \sim 1$ eV, $L \sim 1$ m, and $\boldsymbol{H_0} \sim 5$ T, one finds a value of birefringence $\beta_\Psi \sim 10^{-20}$ – much below the current experimental limits.

Thus, the final range of allowed parameters of millicharge fermions ranges from

$$v_B \lesssim 1 \text{ eV} \quad ; \quad e_\Psi \lesssim 10^{-8} \quad ; \quad q_\Psi < 1 \tag{36}$$

to

$$v_B \lesssim 10^{-10} \text{ eV} \quad ; \quad e_\Psi \lesssim 10^{-14} \quad ; \quad q_\Psi < 1 \tag{37}$$

with the fermion mass $\omega_{\text{lab}} \lesssim m_\Psi < 20$ eV.[8]

We see, that this range of parameters differs from those of Refs. [20, 21], which suggested that the PVLAS experiment can be explained by the existence of millicharged particles with $m_\Psi \sim 0.1$ eV and charges $e_\Psi \sim 10^{-6}$. Such millicharged particles, however, would introduce a distortion of CMB energy spectrum [27]. On the other hand, the millicharged particles with parameters (36)–(37) and vector field with parameters (30) can give rise to effects testable in present and forthcoming experiments (PVLAS, OSQAR, ALPS, LIPPS, BMV, Q&A): shining light through the wall and dichroism and birefringence of light propagating in strong magnetic field.[9]

4 Discussion

In this paper we presented a simple example of a theory in which non-trivial anomaly cancellation between light and heavy sectors gives rise to possible observable effects in optical laboratory experiments. We showed that the structure of the effective action changes at high energies, suppressing the production of light particles. Therefore the stellar constraints can be significantly weaker in this case, allowing for relatively strong optical effects, that may be observed in current or future experiments.

We would like to notice that although we have presented an explicit microscopic theory (19), which leads to the effective theory (16) we are interested in, the actual fundamental theory could be rather different. For example, instead of heavy set of fermions Ξ in the theory (19), the CS term may be generated by different mechanisms in the underlying high-energy theory.

Actually, the low-energy effective action (1) or (2) with the CS coupling can arise easily in D-brane realizations of the Standard Model [43, 29]. Indeed, gauge groups come often in unitary factors containing extra $U(1)$'s. Moreover, the smallness of

[8] The allowed window for m_Ψ can become higher, if future laboratory experiments strengthen bounds of [6].

[9] Note that as eq. (30) shows, taking v_B in the sub-eV range allows to have $M \sim 10^5$ GeV and thus explaining the effect of dichroism, reported by the PVLAS collaboration in 2006 [3]. To avoid the stellar constraints this would require to have $m_\Psi \sim \omega_{\text{lab}} \sim 1$ eV. However, the contribution of these fermions to dichroism at laboratory energies is subleading compared to the corresponding contribution of the CS term (2), which differs this model from the one proposed in [21, 20].

the mass m_B and the CS coupling $\boldsymbol{\kappa}$ of the vector boson B_μ to the photon may be naturally explained in models of large extra dimensions and low string scale [44, 45]. For this, B should propagate in the bulk of extra dimensions, while its mass should arise from localized anomalies due to chiral fermions present for instance in the intersections of the Standard Model branes with the brane extended in the bulk. Assuming for simplicity an homogeneous bulk of volume $v_\perp$ in string units and identifying the axion with the string scales $M \sim M_s$, one has $\boldsymbol{\kappa} \sim 1/\sqrt{v_\perp}$ and $m_B \sim M/\sqrt{v_\perp}$. It follows that $m_B \sim 1$ eV for $M \sim 100$ TeV, in which case $\boldsymbol{\kappa} \simeq 10^{-14}$. On the other hand, the microscopic theory with millicharged fermions is more difficult to accommodate. One could imagine for instance that the hypercharge (or the electric charge) acquires a tiny mixing with the bulk gauge boson B, creating millicharges, although it is not clear how to get a consistent setup with "natural" suppression.

Another example of a theory with non-trivial anomaly cancellation between light and heavy sectors, is in the presence of extra dimensions, discussed in [17, 18]. In this theory a plane wave, propagating in a strong magnetic field $H_x \approx \mathbf{const}$ with polarization parallel to the field, is described by the equation

$$\frac{1}{\Delta(z)}\partial_z\Big(\Delta(z)\partial_z A_x\Big) + \Box\, A_x = \frac{\alpha_{\mathrm{EM}}^2\kappa_0^2\vec{H}^2}{M_5^2\Delta^2(z)}A_x + \mathcal{O}(\kappa_0) \tag{38}$$

This leads to the massive wave equation

$$\Box\, A_x(x) - m^2_{\gamma\, H}\, A_x(x) = 0 \tag{39}$$

where the "magnetic mass"

$$m^2_{\gamma\, H} \sim \alpha_{\mathrm{EM}}\kappa_0|\vec{H}| \tag{40}$$

depends only on 4-dim quantities; it is not suppressed by the scale of the 5-dim theory M_5. The wave equation for perpendicular to the magnetic field component remains the same (massless): $\Box\, A_y(x) = 0$.

This theory is drastically different from the models discussed in this paper (as well as other models, predicting non-trivial effects in strong magnetic fields). Namely, it does not contain any new light degrees of freedom. This means, that the regeneration experiments ("shining light through the wall"), as well as the measurements of dichroism will produce no results in this case. However the theory leads to an ellipticity (birefringence) of the linearly polarized light:

$$\Delta\phi = \frac{m^2_{\gamma\, H}}{2\omega}L \sim \frac{\kappa_0\alpha_{\mathrm{EM}}|\vec{H}|}{2\omega}L \tag{41}$$

Actually, the static ("capacitor") experiment suggested in [18] is also sensitive to the model described in this paper. As different models, predicting non-trivial results in optical experiments behave differently in this static experiment, it provides a good way to distinguish among them.

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